No. of Printed Pages : 4

Sem-III-UG-Math-CC-VI
(Arts & Science)

2021-22

Time - 3 hours

Full Marks - 80

*Answer **all parts** as per instructions.*
Part of each question should be answered continuously.
Figures in the right hand margin indicate marks.
The symbols used have their usual meaning.

PART – I

1. Answer all questions. [1 × 12

(a) In D_4 , find all elements X such that, $X^3 = H$.

(b) Find the inverse of the element under the given operation 13 in Z_{20} .

(c) Give the reason that the set of integers under subtraction is not a group.

(d) Define Trivial subgroup of a group.

(e) How many subgroups of order 4 does D_4 have ?

(f) In a group Z, find $\langle 8,14 \rangle$.

(g) Let 'a' be an element of a group and $|a| = 15$, compute the order of a^3 of G.

(h) Write how many subgroups of the group S_4 .

(i) Find the order of permutations (14).

(j) Define isomorphism of a group.

(k) Define Lagrange's theorem.

(l) Define kernel of a Homomorphism.

PART – II

2. Answer any eight questions. [2 × 8

(a) Write the statement of Cancellation laws of group.

(b) The subset {1, –1} will be subgroup of G = {1, –1, i, –i} under multiplication. Justify.

(c) If H be a non-empty subset of a geoup G, then define H^{-1}.

(d) A cyclic group is abelian. Justify.

(e) Define centre of group G.

(f) If $\phi : G \to \bar{G}$ is an isomorphism, then prove $\phi(e) = \bar{e}$, $e \in G$, $\bar{e} \in \bar{G}$.

(g) Find $U(8) \oplus U(10)$ where U(n), n > 1 denotes the set of positive integers less than n and relatively prime to n.

(h) State third isomorphism theorem.

(i) Define right coset of H in G, where H is a subgroup.

(j) Find the order of Z_{12}.

PART – III

3. Answer any eight questions. [3 × 8

(a) If H and K are subgroups of G, show that $H \cap K$ is a subgroup.

(b) If G is a group, prove that the only element $a \in G$ with $a^2 = a$ is 1.

(c) Find the order of the permutation

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 6 & 1 & 2 & 3 & 4 & 5 \end{bmatrix}.$$

(d) Let G be a group and $H = \{g^2 : g \in G\}$ is a subgroup of G. Prove that H is a normal subgroup of G

(e) Let ϕ be a homomorphism from G to $\bar{G}$. H be a subgroup of G. Then prove if H is abelian, then $\phi(H)$ is abelian.

(f) Prove that identity element in a group is unique.

(g) If H and K are subgroups of a finite group such that $O(H) > \sqrt{O(G)}$, $O(K) > \sqrt{O(G)}$, then $O(H \cap K) > 1$. Prove it.

(h) Find generators of Z_8.

(i) The set of integers is a cyclic group with respect to addition above 1 is its generator. Justify.

(j) If G a finite group and $a \in G$, show that $a^{O(G)} = e$.

PART – IV

Answer all questions.

4. Show that the set G = {0, 1, 2, 3} is not a group under multiplication modulo 4. [7

OR

Define centralizer of a group and show that centralizer of a group forms a subgroup of G.

5. State and prove Two-step subgroup test. [7

OR

Show that the symmetric group S_3 under function composition forms a group.

6. A subgroup H of G is normal in G iff $xHx^{-1} \subseteq H$, for all x in G. Prove it. [7

OR

Show that $Z_{25} \oplus Z_5$ has 24 elements of order 5.

7. If $\phi : G \to \bar{G}$ is a homomorphism, $a \in G$, then prove that $\phi(a^n) = (\phi(a))^n$. [7

OR

State and prove Cayley's theorem for group isomorphism. [7